Physical limits of Communication

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Abstract

We describe recent work with Sanjeev Khanna (U. Penn.) where we explore potential axioms about the mechanics of information transmission with a view to understanding whether continuous signals can carry more information than analog signals.

1998 ACM Subject Classification H.1.1 Systems and Information Theory

Keywords and phrases Analog signals, information capacity, delays

Digital Object Identifier 10.4230/LIPIcs.FSTTCS.2011.4

1 Summary

Year after year, we have seen explosions in the amount of information that we can store on storage devices, and the speed at which we can ship this information around the universe. Are we close to reaching limits imposed by the physics? What are these limits and what axioms do they come from? The study of such questions has primarily been the domain of signal processing and information theory. In this talk we will speak about joint work with Sanjeev Khanna, in which we explore such questions with some discrete and probabilistic modeling.

The fundamental underlying issue is that digital communication is implemented on physical objects. Classical models of the communication power of this physical layer study the information carrying power of "continuous time signals", i.e., real-valued functions over a continuous interval of time. In the absence of sources of uncertainty such signals have "infinite" information carrying capacity for two reasons: (1) At each instant of time, the signal has infinitely many different values; and (2) There are infinitely many instances of time (even in a bounded interval).

The former issue was resolved quite convincingly in the work of Shannon, who said that if there is a additive error at each instant of time (where the error could be a simple Gaussian random variable), then the information carrying power of a signal bounded in energy is a finite number of bits. The latter issue unfortunately appears to be a bit more subtle. Classical signal processing tends to deal with this issue by first looking at the Fourier spectrum of the signal being transmitted, and then placing some restrictions on these. Converting back to time domain, these restrictions miraculously say it is sufficient to sample the signal at discrete time intervals. But what are these restrictions, where did they come from, and are they really distinct from the assumption that the signal should be inferrable from its values at a discrete set of sample points? In this talk we discuss such questions, and then contrast with the more discrete and probabilistic models analyzed in our work [1].

In our work we ask what happens if nature provides us with a collection of N particles, each one of which is capable of transmitting one bit somewhat unreliably. In particular we study the case where this unreliability comes in two forms: *error*, where the bit carried by a particle may flip during transmission, and *delay*, where the particle's arrival time at a

31st Int'l Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS 2011).
Editors: Supratik Chakraborty, Amit Kumar; pp. 4–5
Leibniz International Proceedings in Informatics
LIPICS Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

destination may not correspond exactly to its departure time. In particular we let the delay be a random variable with a variance of one unit of time, and study how much information can the sender communicate in T units of time, The natural method to deal with this uncertain delay would be to transmit roughly N/T particles every, say, two units of time and then the delay variance of one unit of time would not lead to interference from particles transmitted at different time slots. But this would lead to a rate of information transmission of only $T \log(N/T)$ bits. A more "continuous" transmission protocol might allow particles to be transmitted at any point of time in the interval $[0, T]$. Would this enhance the capacity? Surprisingly our (simple) analysis shows that the capacity does grow with such schemes. In particular one can transmit as $T \cdot (N/T)^{\epsilon}$ bits, for some positive ϵ, in T units of time in such settings. Thus we find that using the continuum of time does enhance capacity, up to physical limits imposed by the particular nature of matter.

References

1 Sanjeev Khanna and Madhu Sudan. Delays and the capacity of continuous-time channels. *CoRR*, abs/1105.3425, 2011.