Communication Amid Uncertainty

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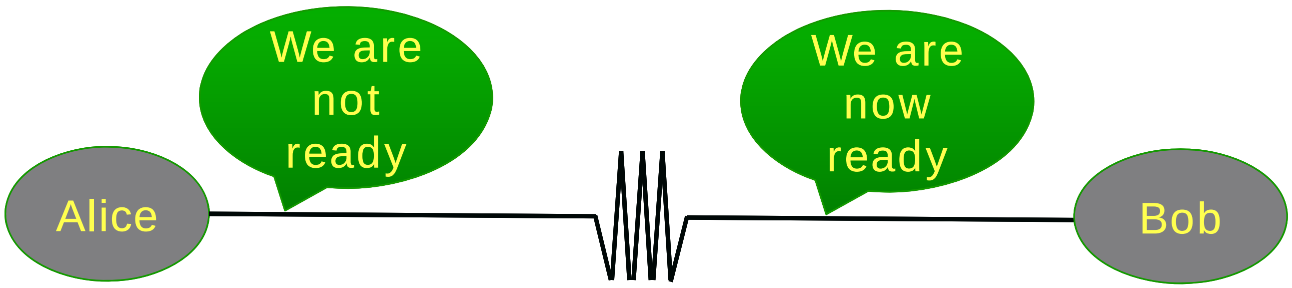
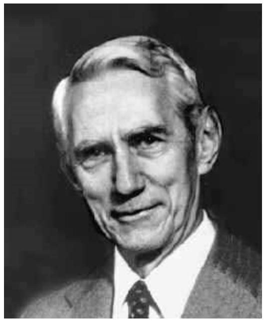
Reliable Communication?

- Problem from the 1940s: Advent of digital age.



- Communication media are always noisy
  - But digital information less tolerant to noise!

(Mathematical) Theory of Communication

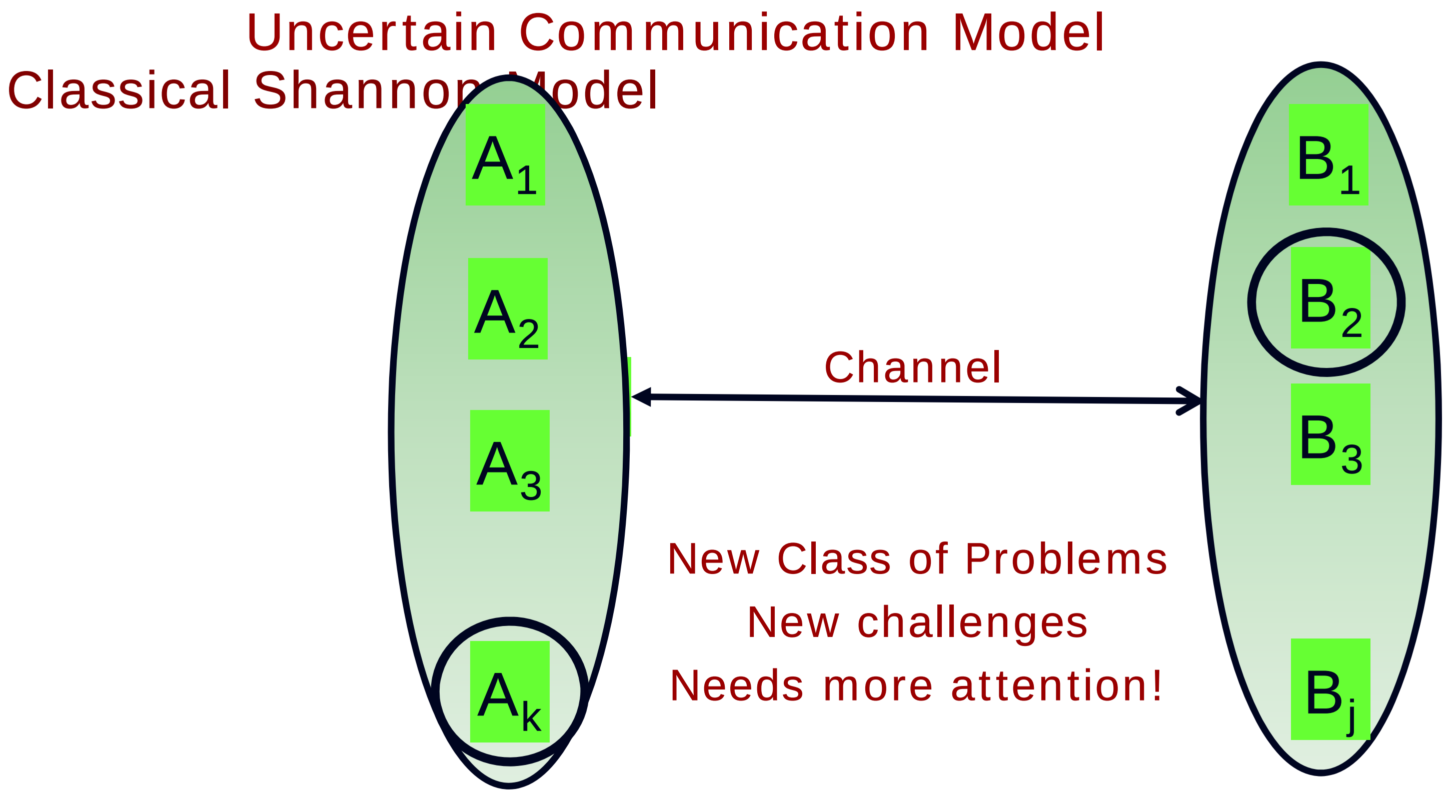


- [Shannon, 1948]
- Questions considered:
  - How to <u>compress</u> information (to eliminate common knowledge of sender and receiver) and minimize #bits communicated?
  - How to <u>detect</u> errors injected by communication channel?
  - How to <u>correct</u> them?
- Fundamental discoveries:
  - "Bit", Entropy, Mutual Information, Coding, Decoding ...
  - Driver of research and technology for 65 years!

Uncertainty in Communication?

- Always an issue … but usually uncertainty in <u>channel</u> of communication.
- Lately, however … Also have to worry about uncertainty of communicating parties about each other.
  - Communicating with printer: What format does it like the document?
  - E.g.: How would you like to archive your family photographs “digitally” when you are uncertain which format will be viewable? Would you compress?
- New class of questions … new solutions needed.
  - Often “natural” communication encounters/solves these problems. How?
- First, a new model.

Modelling uncertainty

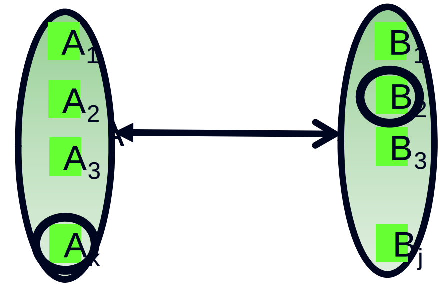


Example 1: Communication with uncertain priors

- In most natural communication, sender and receiver don't understand each other's "knowledge" perfectly.
  - They form estimates, and can get close, or at least get a good idea of how far they are.
    - Sender's "prior" $\neq$ Receiver's "prior"; only close.
  - Yet, they seem to compress communication quite well …
    - e.g., this talk!
  - How?
    - Classical compression schemes break down completely if Sender's "prior" $\neq$ Receiver's "prior";

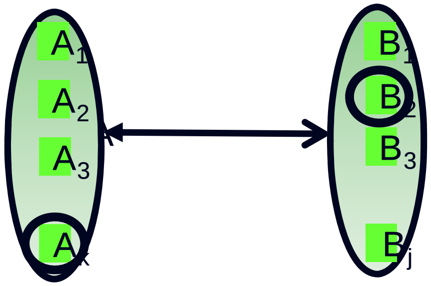
Example: Compression

- How would you compress information if Sender and Receiver don't have a common prior?
  - Message space $\mathbb{M} = \{1, \ldots, N\}$
  - Encoding: $E(P, m)$, P dist. on $\mathbb{M}$, $m \leftarrow_P \mathbb{M}$.
  - Decoding: $D(Q, y)$. (Q dist., $y \in \{0,1\}^*$)
  - Minimize $\mathrm{E}_{m \leftarrow_P \mathbb{M}}[|E(P, m)|]$
  - Need: $D\big(Q, E(P, m)\big) = m$ provided $P \approx Q$



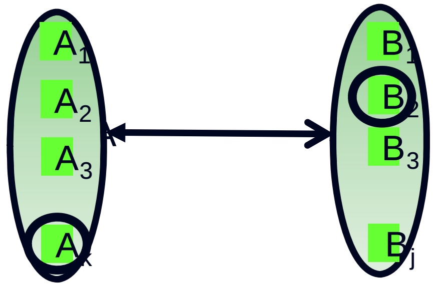
- Explored in
  - [Juba,Kalai,Khanna,S'11]: With randomness.
  - [Haramaty,S'13]: Deterministic.

Flavor of results



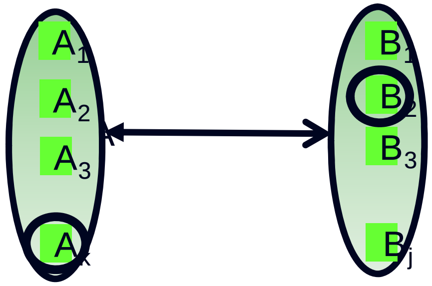
- [JKKS]:
  - Assume sender and receiver have common dictionary
    - Gives sequence of longer and longer words for every message.
    - Each word corresponds to many messages!
    - Encode message m using shortest word w for which m has highest probability under P by large margin.
    - Decode w to message $\tilde{m}$ that has highest prob. under Q.
- Theorem: Compresses to $H(P) + 2\Delta$ bits in expectation, if dictionary is random, and $P \approx_{\Delta} Q$

Misunderstanding and Meaning



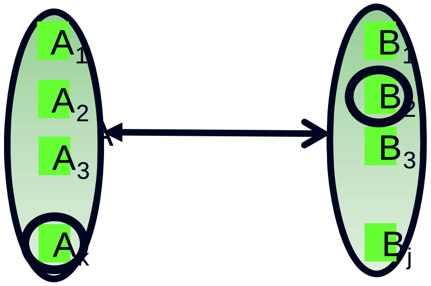
- Bits lead to action
  - How can sender ensure receiver understands instruction and acts accordingly?
    - Incentive?
      - Receiver may not want to follow sender's instructions.
      - Or receiver may not understand ...
- Goal-oriented comm. [GoldreichJubaS.12]
  - Sender must have goal + sense progress.
  - Achievement of goal is "functional" defn. of communicating meaningfully.
  - Sufficient conditions for comm. meaningfully.

Example: Communication as Coordination Game [Leshno,S.'13]



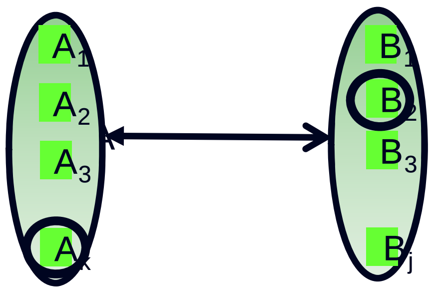
- Two players playing series of coordination games
  - Coordination?
    - Two players simultaneously choose 0/1 actions.
    - "Win" if both agree:
      - Alice's payoff: not less if they agree
      - Bob's payoff: strictly higher if they agree.
  - Knowledge about each other?
    - Don't know each other's strategy exactly.
    - But know "set of reasonable" strategies from which the other players chooses one. How should Bob play?
      - Can he hope to learn? After all Alice is "reasonable"?
      - Specifically: Always coordinate after finite # of initial miscoordinations?

Example: Communication as Coordination Game [Leshno,S.'13]



- Two players playing series of coordination games
  - Coordination?
    - ...
  - Knowledge about each other?
    - "Reasonable"
    - Can Bob learn to coordinate, while being "reasonable"?
- Theorem:
  - **Not Deterministically** (under mild "general" assumptions)
    - Alice may be trying to learn Bob at same time!
  - **Yes, with randomness** (for many "broad" notions of reasonability: computably-coordinatable, poly-time coordinatable etc.)
    - Can break symmetries with randomness.

Conclusions



- Communication, when mixed with computation (intelligence), leads to new mathematical problems.
- Model many aspects of “natural” communication; and many challenges in the digital age.
- Lots of work to be done!

Thank You!

	0	1
0	X,1	Y,0
1	Z,0	W,1

Alice Payoffs: $X \geq Y$; $W \geq Z$