Communication Amid Uncertainty

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Reliable Communication?

- Problem from the 1940s: Advent of digital age.

- Communication media are always noisy
  - But digital information less tolerant to noise!
(Mathematical) Theory of Communication

- [Shannon, 1948]

- Questions considered:
  - How to compress information (to eliminate common knowledge of sender and receiver) and minimize #bits communicated?
  - How to detect errors injected by communication channel?
  - How to correct them?

- Fundamental discoveries:
  - “Bit”, Entropy, Mutual Information, Coding, Decoding ...
  - Driver of research and technology for 65 years!
Uncertainty in Communication?

- Always an issue ... but usually uncertainty in channel of communication.
- Lately, however ... Also have to worry about uncertainty of communicating parties about each other.
  - Communicating with printer: What format does it like the document?
  - E.g.: How would you like to archive your family photographs “digitally” when you are uncertain which format will be viewable? Would you compress?
- New class of questions ... new solutions needed.
  - Often “natural” communication encounters/solves these problems. How?
- First, a new model.
Modelling uncertainty

New Class of Problems
New challenges
Needs more attention!
Example 1: Communication with uncertain priors

- In most natural communication, sender and receiver don’t understand each other’s “knowledge” perfectly.
  - They form estimates, and can get close, or at least get a good idea of how far they are.
  - Sender’s “prior” ≠ Receiver’s “prior”; only close.
- Yet, they seem to compress communication quite well ... 
  - e.g., this talk!
- How?
  - Classical compression schemes break down completely if Sender’s “prior” ≠ Receiver’s “prior”;

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Example: Compression

How would you compress information if Sender and Receiver don’t have a common prior?

- **Message space** $\mathcal{M} = \{1, \ldots, N\}$
- **Encoding**: $E(P, m)$, $P$ dist. on $\mathcal{M}$, $m \leftarrow_P \mathcal{M}$.
- **Decoding**: $D(Q, y)$. ($Q$ dist., $y \in \{0,1\}^*$)
- **Minimize** $E_{m \leftarrow_P \mathcal{M}}[|E(P, m)|]$  
- **Need**: $D(Q, E(P, m)) = m$ provided $P \approx Q$

Explored in

- [Juba,Kalai,Khanna,S’11]: With randomness.
- [Haramaty,S’13]: Deterministic.
Flavor of results

- [JKKS]:
  - Assume sender and receiver have common dictionary
    - Gives sequence of longer and longer words for every message.
    - Each word corresponds to many messages!
  - Encode message $m$ using shortest word $w$ for which $m$ has highest probability under $P$ by large margin.
  - Decode $w$ to message $\hat{m}$ that has highest prob. under $Q$.
- Theorem: Compresses to $H(P) + 2\Delta$ bits in expectation, if dictionary is random, and $P \approx_\Delta Q$
Misunderstanding and Meaning

- Bits lead to action
  - How can sender ensure receiver understands instruction and acts accordingly?
    - Incentive?
      - Receiver may not want to follow sender’s instructions.
      - Or receiver may not understand …
- Goal-oriented comm. [GoldreichJubaS.12]
  - Sender must have goal + sense progress.
  - Achievement of goal is “functional” defn. of communicating meaningfully.
  - Sufficient conditions for comm. meaningfully.
Example: Communication as Coordination Game
[Leshno, S. ’13]

- Two players playing series of coordination games
  - Coordination?
    - Two players simultaneously choose 0/1 actions.
    - “Win” if both agree:
      - Alice’s payoff: not less if they agree
      - Bob’s payoff: strictly higher if they agree.
  - Knowledge about each other?
    - Don’t know each other’s strategy exactly.
    - But know “set of reasonable” strategies from which the other players chooses one. How should Bob play?
      - Can he hope to learn? After all Alice is “reasonable”?
      - Specifically: Always coordinate after finite # of initial miscoordinations?
Example: Communication as Coordination Game
[Leshno, S.’13]

- Two players playing series of coordination games
  - Coordination?
  - ...
  - Knowledge about each other?
    - “Reasonable”
    - Can Bob learn to coordinate, while being “reasonable”?

- Theorem:
  - Not Deterministically (under mild “general” assumptions)
    - Alice may be trying to learn Bob at same time!
  - Yes, with randomness (for many “broad” notions of reasonability: computably-coordinatable, poly-time coordinatable etc.)
    - Can break symmetries with randomness.
Conclusions

- Communication, when mixed with computation (intelligence), leads to new mathematical problems.
- Model many aspects of “natural” communication; and many challenges in the digital age.
- Lots of work to be done!
Thank You!
Alice Payoffs: $X \geq Y; W \geq Z$